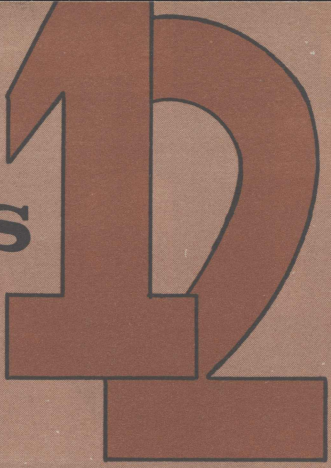
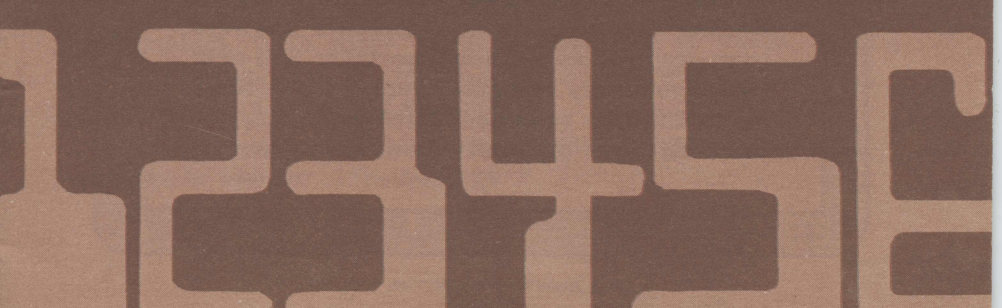
MATH time-savers





1234 MATH time-savers

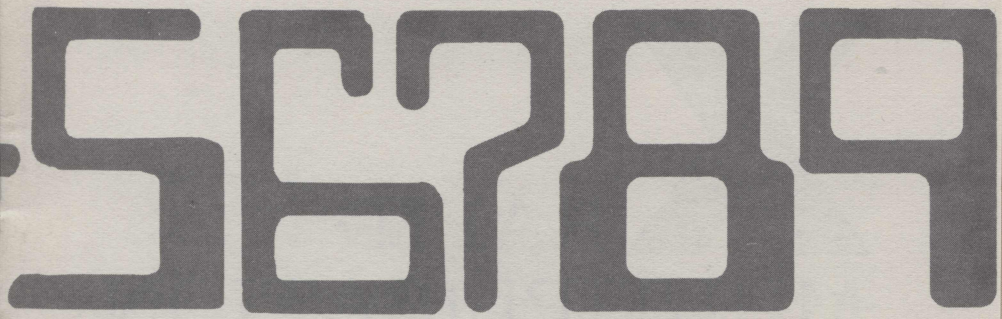
BY WILLIAM C. VERGARA

Mr. Vergara is the author of Mathematics in Everyday Things and many other full-length books in the area of science and mathematics. A contributor of articles to The Book of Knowledge, he is a member of the National Association of Science Writers.

SOMETHING FOR EVERYONE

Whether you are an industrial or office employee, a housewife or sales person, you will find arithmetic shortcuts of value and use to you in this manual. You will be amazed at how simple many of them are. Master those with greatest application to your needs, and you will have powerful tools for finding the quick or approximate answers to many problems. These shortcuts will help you save a very valuable commodity — time. Even in this so-called electronic age, we can't all have our individual computers.

© Good Reading Rack Service, Inc., 1967



Once you have mastered the simple rules in this booklet, arithmetic will become an exciting new game.

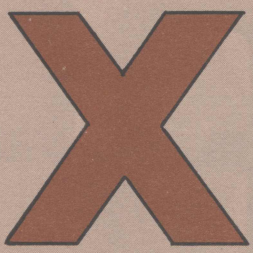
You will be able to do many kinds of problems as fast as a computing machine — often writing the answer as easily as you write your own name. You will be able to add long columns of figures in your head — without resorting to pencil and paper. With a little practice, you will find numbers falling in step like soldiers on parade.

Learning the simple steps in the following pages is as easy as learning the rules of a new game. Even if you often have trouble with mathematics, the results will surprise you and amaze your friends. Whether you are a salesclerk, shopper, office worker or engineer, you will become a magician with numbers.

Nevertheless, the mathematical time-savers that follow are merely a beginning. As each one is mastered, you will probably find other shortcuts coming to mind. Often, for example, you need or want only an approximate answer to a problem. With these rules in your bag of tricks, you will find yourself approximating the answer to difficult problems in a matter of seconds.

Those who want to brush up on multiplication will find a table at the end of the booklet.

So, begin saving time, and happy reckoning!



RAPID MULTIPLICATION

Ask anyone to multiply 18×15 and the odds are he will glance around nervously for pencil and paper. Yet a simple method can provide the answer in seconds: $18 \times 15 = 270$.

But before we learn this first trick of the multiplication trade, it will help to define a few necessary terms.

1) The **multiplicand** is a number that is multiplied by another, the "18" in the example given below.
2) The **multiplier** is the number by which another is multiplied, the "15" in the example given below.
3) The **product** is the answer obtained when two numbers are multiplied, the "270" in the example.

18 = multiplicand

x15 = multiplier

90

18

270 = product

4) The **units** figure or numeral of a number represents the number of "ones" or "units" in the number; it is the figure to the far right of a number, the "0" in the product above. Similarly, the **tens** figure is just to the left of the units figure and gives the number of tens in the number. The next numeral to the left is the **hundreds** figure, then the **thousands** figure and so on.

In school, most of us learned the multiplication table through $12 \times 12 = 144$. A few more venturesome souls even reached the stratospheric 13's. But the following simple rule automatically extends our mastery of the multiplication table through the teens, enabling us to multiply any two numbers up to 19×19 with lightning-like speed.

Multiplying by the Teens

Problem: $18 \times 15 = ?$

Rule:

(a) Add either number (the multiplier or the multiplicand) to the unit figure of the other, and annex a zero;

(b) to this add the product of the unit figures of both original numbers. Sound complicated? Not really — here's how it works in practice:

Example: $18 \times 15 =$

(a) $18 + 5 = 23$, or, 230 (annexing a zero)

(b) $8 \times 5 = 40$, so, $230 + 40 = 270$ (answer)

In using this rule, your thought processes run something like this: "Add 18, (the first number), to 5, (the unit figure of the second number), and get 23; annex a zero and it becomes 230; multiply 8×5, (the two unit figures), which gives 40; add 230 to 40 to get the answer, 270.

Try another example: $15 \times 14 = ?$ ($15 + 4 = 19$; 190; $5 \times 4 = 20$; $190 + 20 = 210$).

If you run through a few examples, you will be amazed at how easy it is to mentally multiply the teens, and your friends will be amazed at your new-found mathematical wizardry!

Squaring a Number that Ends in 5

Problem: $35 \times 35 = ?$

Rule:

(a) Multiply the 5's in the units column and write down 25 for the last two figures of the product;

(b) add 1 to the first 3 (or other numeral) in the tens column which makes 4 and then;

(c) multiply this figure (4) by the other tens column figure (3) to get 12 which is written before the 25 to give the answer.

Example: $35 \times 35 =$

(a) $5 \times 5 = 25$

(b and c) $4 \times 3 = 12$

(Answer) 1225

Multiplying any number by 25 or 50

Problem: $219 \times 25 = ?$

Rule:

First of all, keep in mind that $25 = \frac{100}{4}$ So, to multiply by 25 merely add two 0's to the multiplicand and divide by 4.

Example: $219 \times 25 =$

$21900 \div 4 = 5{,}475$ (Answer)

Problem: $219 \times 50 = ?$

Rule:

Because $50 = \frac{100}{2}$, merely add two 0's to the multiplicand and divide by 2.

Example: $219 \times 50 =$

$21900 \div 2 = 10{,}950$ (Answer)

Multiplying with Complements

Problem: $93 \times 95 = ?$

Rule:

The complement of a number is the figure required to bring it up to 100; so 7 is the complement of 93, and 5 is the complement of 95.

(a) Multiply the two complements ($7 \times 5 = 35$) and write down the product (35) for the final portion of the answer;

(b) from either number, subtract the complement of the other number (say $93 - 5 = 88$, or $95 - 7 = 88$) and write down the remainder (88) as the first portion of the answer.

Example: $93 \times 95 =$

(a) $7 \times 5 = 35$

(b) $93 - 5 = 88$

8835 (Answer)

Multiplying with Supplements

Problem: $106 \times 105 = ?$

Rule:

The supplement of a number is the figure by which it exceeds 100. So, 6 is the supplement of 106 and 5 is the supplement of 105.

(a) Multiply the two supplements ($6 \times 5 = 30$) and write the product (30) as the last part of the answer;

(b) add the supplement of either number to the other number ($106 + 5 = 111$), and write the sum (111) as the first figures of the answer.

Example: $106 \times 105 =$

(a) $6 \times 5 = 30$

(b) $106 + 5 = 111$

11130 (Answer)

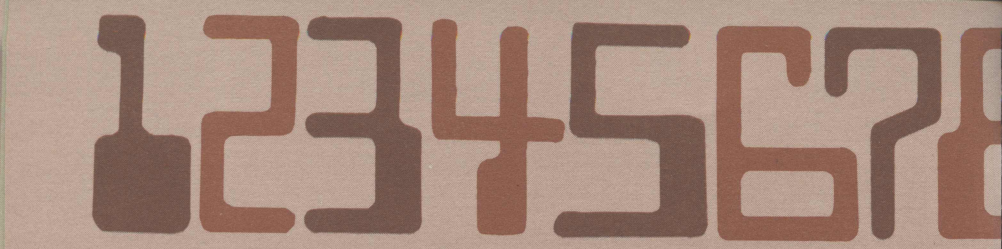
Multiplying by Any Number Formed of Nines

Problem: $536 \times 999 = ?$

Rule:

(a) Add one zero to the multiplicand for each nine in the multiplier;

(b) from this number subtract the multiplicand to get the answer.



Example:

$536 \times 999 =$

(a) 536,000 (annexing three 0's)

(b) $\underline{-\ 536}$

535,464 (Answer)

Multiplying by 11; 101; 1001; 10001; etc.

Problem:

$78 \times 1001 = ?$

Rule:

(a) Count the number of figures in the multiplier, (4, above); subtract 1 from that number, $(4 - 1 = 3)$; and add that many 0's to the multiplicand;

(b) to that number add the multiplicand.

Example:

$78 \times 1001 =$

(a) 78,000 (annexing three 0's)

$\underline{+\ 78}$

78,078 (Answer)

Multiplying Any Number by 24 or 49

Problem:

$17 \times 24 = ?$

Rule:

(a) Multiply by 25 as explained earlier;

(b) from that number subtract the multiplicand.

Example:

$17 \times 24 =$

(a) $1700 \div 4 = 425$

(b) $\underline{-17}$

408 (Answer)

Problem:

$17 \times 49 = ?$

Rule:

(a) Multiply by 50 as explained earlier;

(b) from that number subtract the multiplicand.

Example:

$17 \times 49 =$

$1700 \div 2 = 850$

$\underline{-17}$

833 (Answer)

Multiplying Any Number by 26 or 51

Problem:

$32 \times 26 = ?$

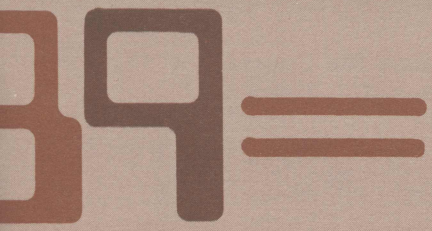
Rule:

(a) Multiply by 25 as explained earlier;

(b) to that number add the multiplicand.

Example:

$32 \times 26 =$



(a) $3200 \div 4 = 800$
(b) $+32$
832 (Answer)

Problem:
$32 \times 51 = ?$

Rule:
(a) Multiply by 50 as explained earlier;
(b) to that number add the multiplicand.

Example:
$32 \times 51 =$
(a) $3200 \div 2 = 1600$
(b) $+32$
1632 (Answer)

Multiplying Any Number by 125

Problem:
$334 \times 125 = ?$

Rule:
(a) Annex three 0's to the multiplicand;
(b) divide that number by 8.

Example:
$334 \times 125 =$
$334000 \div 8 = 41{,}750$ (Answer)

Multiplying Any Number by 11

Problem:
$612 \times 11 = ?$

Rule:
(a) Write the units figure of the multiplicand as the units figure of the answer;
(b) to get each succeeding figure, merely add each figure of the multiplicand to the neighbor on its right;
(c) write the left-hand figure of the multiplicand as the left-hand figure of the answer.

Example:
$612 \times 11 =$
(a) 2 (units figure of multiplicand)
(b) 32 (1 + 2 = 3)
(b) 732 (6 + 1 = 7)
(c) 6732 (6 is the left hand figure)

The following example shows how this method works when you have to "carry" a figure.

Second Example:
$475 \times 11 =$
(a) 5
(b) $^{1}25$ (7 + 5 = 12, write 2, carry 1)
(b) $^{1}225$ (4 + 7 = 11, 11 + 1 carried = 12, write 2 carry 1)
(c) 5225 (4 + 1 carried = 5)

Multiplying Any Number by 12

Problem:

$654 \times 12 = ?$

Rule:

(a) Double the right-hand figure of the multiplicand and write it as the right-hand figure of the answer;

(b) double each succeeding figure and add it to its right-hand neighbor;

(c) the left-hand figure of the multiplicand (plus anything carried) is the left-hand figure of the answer.

Example:

$654 \times 12 =$

(a) 8 ($4 \times 2 = 8$).

(b) $^{1}48$ ($2 \times 5 = 10$, $10 + 4 =$ 14; write 4, carry 1)

(b) $^{1}848$ ($2 \times 6 = 12$, $12 + 5 =$ 17, $17 + 1$ carried $=$ 18; write 8, carry 1)

(c) 7848 ($6 + 1$ carried $= 7$)

Checking Multiplication by the Lightning Method

A simple and speedy way to check the accuracy of multiplication is based on "casting out nines." It is also called the method of unitates.

The unitate of a number is the sum of the figures that make up the number reduced to one figure. To illustrate, consider the number:

347653

adding the figures $3 + 4 + 7 + 6 + 5 + 3$ gives

28

adding these figures $2 + 8$ gives

10

adding these figures $1 + 0$ gives

1

so 1 is the unitate of 347653.

Taking two more examples,

503792 → 26 → 8 =

unitate of 503792

437965 → 34 → 7 =

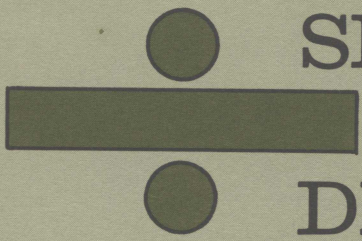
unitate of 437965

The method of using unitates in checking multiplication can best be understood with the help of an example:

	Unitate		Unitate
432	→ 9	$9 \times 7 = 63$ →	9
x331	→ 7		
432			
1296			
1296			
142992	→ 27	→	9

In this example the unitate of the multiplicand is 9 ($4 + 3 + 2 = 9$), and the unitate of the multiplier is 7 ($3 + 3 + 1 = 7$). Multiplying these unitates we have $9 \times 7 = 63$ and the unitate of 63 is $6 + 3 = 9$.

Looking at the product, we see that the sum of its figures is 27 and the unitate of 27 is $2 + 7 = 9$. Since the two unitates are equal, (9), the work is probably correct.



SHORTCUTS IN DIVISION

It is sometimes desired to find a divisor that "goes into" a number evenly, with no remainder. Here are a few rules that save a great deal of effort in finding such divisors.

A. An even number can always be divided by 2.

B. When the sum of the figures forming a number can be divided by 3, the number itself can be divided by 3.

Example:

474 ($4 + 7 + 4 = 15$; $15 \div 3 = 5$)

C. When a number ends in two 0's, or the last two figures can be divided by 4, the number can be divided by 4.

Examples:

3200 (ends in two 0's)

868 ($68 \div 4 = 17$)

D. When a number ends in 0 or 5, the number can be divided by 5.

Examples:

3200 (ends in 0)

775 (ends in 5)

E. When a number is even and the sum of the figures forming it can be divided by 3, the number can be divided by 6.

Example:

1284 ($1 + 2 + 8 + 4 = 15$, and $15 \div 3 = 5$)

F. When the sum of the odd place figures of a number minus the sum of the even place figures can be divided by 7, often the number can be divided by 7.

Even Place
↓ ↓
2 2 7 2 9
↑ ↑ ↑
Odd Place

$18 - 4 = 14$; $14 \div 7 = 2$

G. When the last three figures of a number can be divided by 8, the number can be divided by 8.

Example:

1796824 ($824 \div 8 = 103$)

H. When the sum of the figures in a number can be divided by 9, then the number can be divided by 9.

Example:

33498 ($3 + 3 + 4 + 9 + 8 = 27$ and $27 \div 9 = 3$).

I. When a number ends in 0, the number can be divided by 10.

J. When the sum of the odd place figures minus the sum of the even place figures of a number can be divided by 11, then the number can be divided by 11.

Example:

Even Place
↓ ↓
8 6 4 1 6
↑ ↑ ↑
Odd Place

(8 + 4 + 6 = 18; 6 + 1 = 7, 18 − 7 = 11 and 11 ÷ 11 = 1)

K. When the sum of the figures of a number can be divided by 3 **and** each of the last two figures can be divided by 4, then the number can be divided by 12.
8148 (8 + 1 + 4 + 8 = 21; 21 ÷ 3 = 7, 4 ÷ 4 = 1 and 8 ÷ 4 = 2)

Dividing Any Number by 10; 100; 1,000; etc.

Problem:
3723 ÷ 100 = ?

Rule:
Merely move the decimal point to the left by as many places as there are 0's in the divisor.

Examples:
3723 ÷ 100 = 37.23
98617 ÷ 10,000 = 9.8617

Dividing a Number by 25 or 50

Problem:
4600 ÷ 25 = ?

Rule:
Drop two 0's and multiply by 4.

Example:
4600 ÷ 25 =
46 × 4 = 184 (Answer)

Problem:
4600 ÷ 50 = ?

Rule:
Drop two 0's and multiply by 2.

Example:
4600 ÷ 50 =
46 × 2 = 92 (Answer)

Dividing a Number by a Number Formed of Factors (Where There Is No Remainder)

Problem:
12775 ÷ 35 = ?

Rule:
5 × 7 = 35, so 5 and 7 are factors of 35. Divide the dividend (12775) by one of the factors (5). Then divide the quotient (2555) by the other factor (7). The second quotient (365) is the answer.

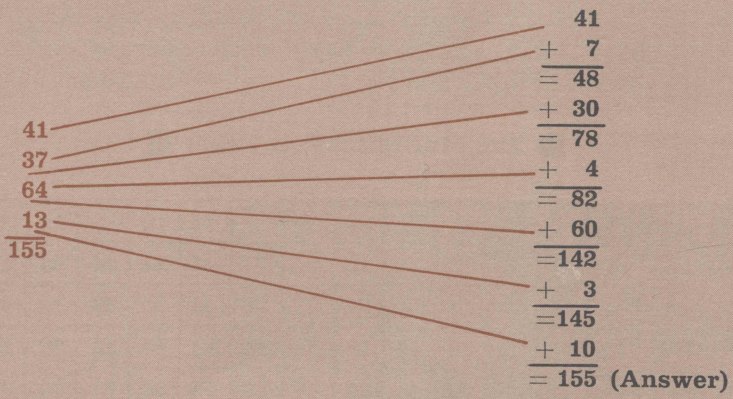
Example:

```
5 | 12775
  7 | 2555
      365 (Answer)
```

In other words, the problem is done by two short divisions instead of one long division.

SIMPLE ADDITION

Have you ever wished you could add up columns of figures mentally without being tied to pencil and paper? With the following simple procedure you can actually come up with a sum before the store clerk. In fact, with a little practice, you can usually call out the answer just as he or she writes down the last figure. Here's how it's done.



Take the top number (41) and mentally add the units figure of the next lower number (7) to it (41 + 7 = 48); then make the tens figure (3) into 30 (its true value) and add it to the first sum (48 + 30 = 78); next add the units figure of the next lower number (78 + 4 = 82); then make the tens figure (6) into 60 and add it to the previous sum (82 + 60 = 142); now add in the units figure (3) to give (142 + 3 = 145); and finally the last tens figure (1) becomes (10) to give (145 + 10 = 155).

By adding mentally, the operations shown on the right-hand column do not appear, and only the successive sums show up in your mind's eye: 41-48-78-82-142-145-155. Of course, you can start with the bottom number and add up just as easily.

This method of simultaneous multi-column addition also works for three columns of numbers:

$ .75
4.20
1.95
2.37
.25

$9.52

Beginning at the top we mentally have the following sums: 75 - 75 - 95 - 495 - 500 - 590 - 690 - 697 - 727 - 927 - 932 - 952 which is the answer, $9.52. Working from the bottom we have 25 - 32 - 62 - 262 - 267 - 357 - 457 - 457 - 477 - 877 - 882 and $9.52.

The Multiplication Table

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225

COMBINED ADDITION & SUBTRACTION

Have you ever had to add up two or more numbers and then subtract their sum from another number? There's a way to do such problems in one simple operation. To illustrate, suppose you had $492.76 in the bank and then drew checks of $3.80, $16.50, $7.69 and $12.16 against the original balance. How much money would be left in the checking account?

Begin by writing the original balance on top, with each of the check amounts in a column beneath it as follows:

$492.76	Original balance
3.80	Check #1
16.50	Check #2
7.69	Check #3
$^{2\ 2\ 1}$12.16	Check #4
452.61	New balance

Then follow these steps:

(a) Add up the right hand (units) column of the checks drawn, (6 + 9 + 0 + 0 = 15), making 15.

(b) From the "15," carry the 1 by jotting it down at the bottom of the next column to the left.

(c) Subtract the 5 (of the "15"), from the 6 of the original balance and write 1 as the units figure of the new balance.

Next, follow the same steps for the tens column. The 1 carried + 1 + 6 + 5 +8 = 21; carry the 2; and subtract 1 from the 7 in the original balance to get 6. Write 6 as the tens figure of the new balance. Following along, 2 + 2 + 7 + 6 + 3 = 20; carry 2; 2 − 0 = 2. Write 2 as the next figure of the answer. Finally, 2+ 1 + 1 = 4; 49 − 4 = 45; so, 4 and 5 are the last two figures of the answer, $452.61.

IR-68-9

LITHO IN U.S.A.